LORENTZ TRANFORM IN THE TWENTY-FIRST CENTURY

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If $\mathcal{L}$ is a Lorentz matrix, so is $-\mathcal{L}$. If $\mathcal{L}$ reverses momentum keeping energy constant, then $-\mathcal{L}$ reverses energy keeping momentum constant, both reversing the velocity v. If $\mathcal{L}$ is expressed as the square of a matrix L, the Lorentz matrix L brings the particle to the rest system and then to $-v$. If $-\mathcal{L}$ is expressed as the square of a matrix, then that matrix with imaginary elements brings the particle to rest but with imaginary mass and then to negative energy. That is

$$\mathcal{L}(v) = L^2 = \begin{bmatrix} \frac{1+v^2}{1-v^2} & \frac{-2v}{1+v^2} \\ \frac{-2v}{1+v^2} & \frac{1+v^2}{1-v^2} \end{bmatrix}, \quad \text{where} \quad L = \begin{bmatrix} \frac{1}{\sqrt{1-v^2}} & \frac{-v}{\sqrt{1-v^2}} \\ \frac{-v}{\sqrt{1-v^2}} & \frac{1}{\sqrt{1-v^2}}. \end{bmatrix}$$

$$-\mathcal{L} = (iL)^2,\ \mathcal{L}(v) = L^2(v) = L(V),\ \text{where}\, V = \frac{2v}{1+v^2},\ v < 1,\ V < 1.$$

Note that

$$\mathcal{L}(\frac{1}{v}) = \mathcal{L}(v) \quad L(\frac{1}{v}) = -iL^*, \quad v < 1,\ V < 1,$$

where

$$L^* = L \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad L^2 = (L^*)^2.$$

L^* is a semi-Lorentz matrix reversing the difference of squares.

What is the physical meaning of L^* and iL^*?

The properties of iL and L^* are connected to the striking features of the velocity transformation formula in which either all three velocities are less than unity, or two of then greater than unity and one less than unity. This has been clarified in the author's 'New Rod Approach to Special Relativity" which explains the distinction between space-like and time-like intervals by a Lorentz Transformation with velocity parameter less than unity. This implies that mathematically a time-like interval can be converted to a space-like interval, or vice-versa by a Lorentz Transform with a velocity parameter greater than unity like iL^*. Note that iL^* preserves the difference of squares while L^* reverses the difference. Both yield velocities greater than unity which is characteristic of space-like intervals! For iL^* yields imaginary space-time coordinates while L^* yields real coordinates both with ratios which are real and greater than unity! Mathematically a space-like interval implies that space and time may be pure real or pure imaginary while the ratio is greater than unity.

The Rod Approach deals with the transformation of space-like to space-like or time-like to time-like coordinates by a Lorentz transform involving a velocity parameter less than unity. Is this related to the Feynman propagator and intermediate states of Dirac "off the energy shell"?

This is an intrusion into the hallowed domain of Dirac and Feynman. Nobel Laureate Chandrasekhar told me frankly "some simple problems which cannot be solved are either ignored or forgotten!" A beginning was made by the author and independently by A. Schoenberg (Brazil) by splitting the propagator into real positive and negative energy parts. Will a new entrant to theoretical physics of the 21st century take up the challenge?