Preface

Srinivasa Ramanujan is one of the greatest mathematicians in history. What makes Ramanujan unique is the spectacular and surprising aspect of almost every one of his discoveries and the manner in which he arrived at his results. Ramanujan's life story is awe inspiring and he has the adulation the world over not just of professional mathematicians but of students and lay persons as well. Indeed one of the best ways to attract bright young students to mathematics is to expose them to the enchanting world of Ramanujan's mathematics.

The Ramanujan Centennial in December 1987 was an occasion when mathematicians around the world gathered in India to pay homage to this singular genius. It was an appropriate time to discuss the significance of his contributions and to consider the ways in which his work would have impact on future research. The Ramanujan Centennial had a major influence on my academic life in ways more than one. I organized an international conference at Anna University in Madras during Dec 19-21, 1987, and spoke on my work on Probabilistic Number Theory - a subject whose origins can be traced to a fundamental paper that Ramanujan wrote with Hardy in 1917. But my attention was drawn to various aspects of Ramanujan's mathematics that directly emanate from his notebooks. In particular, in 1987, I decided to change the focus of my research from classical analytic number theory to the theory of partitions and q-hypergeometric series. I also decided that starting from the Ramanujan Centennial, I would write articles about Ramanujan of general interest and publish them regularly in newspapers and magazines in order to reach a wide audience. I used to lecture regularly about number theory in general and Ramanujan's mathematics in particular in high schools in Madras, but by writing these popular articles, I felt I could reach an even wider audience. This book contains the collection of all such articles I wrote since the Ramanujan Centennial in 1987.

In order to properly understand the significance of Ramanujan's work or to get an idea of the place he occupies in the world of mathematics, we need to compare his life and work with those of other mathematical luminaries whose life and contributions have things in common with Ramanujan. Thus after writing the opening article "Ramanujan - an estimation" in 1987 for the Centennial, and following it with an article "Ramanujan - the second century" in which I discuss briefly the impact that various aspects of his research will have in the following decades, I wrote a series of articles on the lives and works of mathematical giants who had strong links with Ramanujan. This is the first group of articles in this book and they all appeared in "The Hindu" India's National Newspaper, as birthday tributes to Ramanujan annually in December. I felt that these articles would not only be useful to inform the lay public about the many great mathematicians in history and the impact of their contributions, but that one could appreciate Ramanujan better by studying him in comparison with other eminent mathematicians. The Hindu sometimes modified the titles of my articles, as well as the text in some places. The articles are published here with their original titles I had, and with my original text in full. I will now briefly describe why I chose to write about certain mathematicians.

G. H. Hardy, Ramanujan's mentor, has said that for sheer manipulative ability, Ramanujan can be compared with Euler and Jacobi. Also there is much in common regarding the kind of mathematics Euler, Jacobi, and Ramanujan worked on. Thus the separate ar-

1

ticles on Euler and Jacobi not only describe their lives and achievements, but also compare their mathematical methods and results with those of Ramanujan.

In Hardy's own words, the British mathematician L. J. Rogers was not unlike Ramanujan both in talent and in the kind of infinite series identities he investigated. Indeed Rogers had proved (what is now termed as) the Rogers-Ramanujan identities about 15 years before Ramanujan discovered them. And Ramanujan's rediscovery of Roger's work was actually responsible for Rogers' belated recognition by the British mathematical community. Thus the opening article on mathematicians is the one on Rogers.

Major P. A. MacMahon, Hardy's mathematical assistant, studied the Rogers-Ramanujan identities combinatorially. He was the one who verified the celebrated Hardy-Ramanujan formula for the partition function. MacMahon was stationed briefly in Madras, India while serving in the British army. In view of these strong links with Ramanujan, an article on MacMahon is included.

The theory of partitions can be broadly classified into four eras: (i) the era of Euler, the founder, (ii) the era of Sylvester who improved significantly on the results of Euler by using combinatorial methods, (ii) the era of Ramanujan who transformed the subject gloriously, and (iv) the modern era. So I felt that an article on Sylvester would be most appropriate.

G. H. Hardy said that the real tragedy with Ramanujan's life was not his early death at the age of 32, but that during his most formative years, Ramanujan was sidetracked due to lack of formal training. Hardy noted that the best mathematics is done at a young age. He cited Evariste Galois and Neils Henryk Abel as mathematicians who died very young but had made monumental contributions. Thus in two separate articles, the lives and works of Abel and Galois are discussed. The article comparing Abel and Ramanujan was written shortly after the Abel Prize in mathematics was launched by the Norwegian Academy of Science and Letters.

The proof of Fermat's Last Theorem in 1994 created a sensation the world over since it settled a three hundred year old conjecture. I noted that there were some links between Ramanujan and Fermat, and so in early 1995 I wrote an article comparing Fermat and Ramanujan.

The passing away of Paul Erdös in 1996 marked the end of a great era. Erdös was the most prolific mathematician in history and influenced the academic lives of many including me. His path breaking work on probabilitic number theory was inspired by the seminal 1917 paper of Hardy and Ramanujan. Thus the December 1996 article is a memorial tribute to both Erdös and Ramanujan.

No account of Ramanujan is complete without a discussion of his mentor G. H. Hardy, and so an article on Hardy is included in this series. Hardy's famous collaborator J. E. Littlewood studied Ramanujan's two letters of 1913 along with Hardy. It was Littlewood who said that every number is a personal friend of Ramanujan! Thus an article on Littlewood follows the article on Hardy.

The great German algebraist Issai Schur had independently proved the Rogers-Ramanujan identities and saw partition theoretic extensions of them. These identities are sometimes called the Rogers-Ramanujan-Schur identities. Thus my penultimate article on mathematicians is on Schur.

2

Robert Rankin, the Scottish mathematician, was one who knew Hardy, and lived in our time as well. He died just a few years ago. It was he who collected all the papers in G. N. Watson's office (after Watson's death) and sent them to the Wren Library in Trinity College, Cambridge, for preservation. Unknowingly, the last manuscript of Ramanujan (which Watson was studying) was in this collection, and so was placed in the Watson estate at the Wren Library and forgotten. This is what George Andrews unearthed as the Lost Notebook in 1976. Thus the final article on mathematicians is on Rankin.

Naturally, from time to time, in addition to writing about mathematicians, I wrote about certain aspects of Ramanujan's mathematics. These also appeared in The Hindu and form the second group of articles in this book.

Ramanujan's spectacular series for pi are among those used in present day computer calculations of the digits of pi; the first article in the second group is on the history of pi and on Ramanujan's work on this fundamental mathematical constant.

Some of Ramanujan's most significant contributions are in the theory of partitions, an area he gloriously transformed with his magic touch. The article "Ramanujan and partitions" discusses some of his most startling results and their far reaching impact.

In 2005, Manjul Bhargava and Jonathan Hanke solved a fundamental problem on universal quadratic forms that was raised by Ramanujan. Bhargava presented this solution as the Ramanujan Commemmoration Lecture at a conference in SASTRA University in 2005 that I helped organize - the conference at which he and Kannan Soundararajan were awarded the First SASTRA Ramanujan Prizes. So I sent a report to The Hindu about this conference and on the solution of this problem of Ramanujan. This is the final article in the second group in this book.

Periodically I was asked to review various books on Ramanujan. The first was an invitation to review "The Lost Notebook and Unpublished Papers" of Ramanujan published by Narosa that was released on Ramanujan's 100-th birthday, December 22, 1987, by India's Prime Minister Rajiv Gandhi and handed over to Professor George Andrews, who unearthed Ramanujan's Lost Notebook at the Wren Library in Cambridge and who wrote a charming introduction to the Narosa publication; my review of this book appeared in The Hindu in January 1988. Next was a review of Robert Kanigel's famous biography of Ramanujan entitled "The man who knew infinity"; my review appeared in The American Scientist in 1992. Finally I was invited by the American Mathematical Monthly to review the two prize winning books by Bruce Berndt and Robert Rankin entitled "Ramanujan - letters and commentary" and "Ramanujan - essays and surveys". All these reviews are the in third group of articles in this book. In this third group, I have included a short review of "Partitions - a play on Ramanujan" that I wrote for The Hindu in May 2003; I had the pleasure of seeing this play along with George Andrews at the Aurora Theatre in Berkeley. I was so impressed with the play, that I immediately sent a review to The Hindu.

Finally, during the Ramanujan Centennial, I decided that I should create something which will not only be a permanent memorial to Ramanujan but also continue to develop his mathematical contributions in the context of current research. Thus I decided to launch The Ramanujan Journal that is devoted to all areas of mathematics influenced by Ramanujan. My proposal to launch this journal received support from eminent mathematicians worldwide many of whom either served or are now serving on the editorial board. The

3

actual process of launching journal takes time. The first issue of this journal came out in 1997. This journal which was originally launched by Kluwer and now published by Springer, has established itself has one of the major journals. My service to the profession as editor of The Ramanujan Journal naturally led me to be invited by SASTRA University in Kumbakonam, Ramanujan's hometown, to organize their annual conferences relating to Ramanujan's mathematics starting in 2003. Two years later, at my suggestion, SASTRA University generously launched the SASTRA Ramanujan Prize - an annual prize of $10,000 given to very young mathematicians for outstanding contributions to areas influenced by Ramanujan. This prize is now recognized as one of the top mathemmatical prizes in the world. The final group of articles concern my efforts regarding the Ramanujan Journal, the SASTRA conferences, and the SASTRA Ramanujan Prize.

Now that we are celebrating Ramanujan's 125-th birthday in December 2012, we are reflecting on the progress achieved since the Centennial. The time seems appropriate to assemble these articles to provide an idea of the place Ramanujan occupies in the world of mathematics.

Before writing these articles, I used to send drafts of each of them to the Trinity of Ramanujan's Mathematics - Professors George Andrews (Penn State University), Richard Askey (University of Wisconsin) and Bruce Berndt (University of Illinois, Urbana) for their comments. I benefited immensely from their criticism and suggestions. In particular, I have had long discussions with Professor George Andrews about the themes and contents of these articles. I am grateful that this towering figure in the world of Ramanujan's mathematics has written a foreword to this book.

I am most grateful to Mr. N. Ram, Editor-in-Chief of The Hindu, and Mr. N. Ravi, Editor of The Hindu, for generously allocating space in their leading newspaper, and publishing these articles annually since 1987. I am thankful to The Hindu, The American Mathematical Monthly, The Focus Magazine, and the American Scientist, for giving permission to reproduce these articles in this book. The reader has to note that these articles appeared at different points in time and in different venues, and so there will be overlap when these are read collectively in this book.

When I decided to take to a research career in mathematics in 1975, it was my father Professor Alladi Ramakrishnan who told me that in addition to doing research, I should be active in communicating mathematics to school students and to the general public. Thus it was with his encouragement and advice that I wrote these articles. Had he been alive today, he would so happy to see this book published. Equally happy to see these articles collectively published are my mother Mrs. Lalitha Ramakrishnan and my wife Mathura and I appreciate their constant support of all my efforts.

Finally I thank Elizabeth Loew and Shamim Ahmad of Springer for their interest in publishing this collection of articles for the 125-th birth anniversary celebrations of Srinivasa Ramanujan.

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