

# RAMANUJAN'S PLACE IN THE WORLD OF MATHEMATICS\*

Essays providing a comparative study

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## Abstracts of Essays

### 1. Ramanujan: An estimation

**Abstract:** *Srinivasa Ramanujan, one of greatest mathematicians in history, made path-breaking contributions without any formal education, and undeterred by poverty and hardships. Ramanujan's accomplishments are compared with those of certain mathematical luminaries who in their own way faced tremendous difficulties in life, yet made revolutionary contributions.*

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### 2. Ramanujan: The second century

**Abstract:** *Srinivasa Ramanujan's spectacular discoveries revealed surprising connections between apparently unrelated topics, and provided food for thought for mathematicians of subsequent generations. This article describes how Ramanujan's results and ideas will continue to influence research in the century following his centenary in areas such as mock theta functions, congruences for partition functions and coefficients of modular forms,  $q$ -hypergeometric identities, special functions, mathematical physics, and computer algebra.*

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### 3. L. J. Rogers: A contemporary of Ramanujan

**Abstract:** *The British mathematician L. J. Rogers had talents similar to Ramanujan in the theory of  $q$ -hypergeometric series and had discovered and proved the Rogers-Ramanujan identities about two decades before Ramanujan discovered them. In this article the life and mathematical contributions of Rogers are described and the story told of how Ramanujan in England accidentally came across certain papers of Rogers, and the recognition Rogers received after Ramanujan's rediscovery of his work.*

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### 4. P. A. MacMahon: Ramanujan's distinguished contemporary

**Abstract:** *P. A. MacMahon, a noted combinatorialist, wrote a famous treatise on Combinatory Analysis in which he included a discussion of the combinatorial significance of the Rogers-Ramanujan identities, that neither Rogers nor Ramanujan emphasized. This article describes MacMahon's life as a Major in the British Army stationed in India and later as an assistant to G. H. Hardy of Cambridge University. MacMahon's mathematical connections with Ramanujan are not just due to the Rogers-Ramanujan identities, but also due the table of partitions he compiled in order to verify the famous Hardy-Ramanujan asymptotic formula for the partition function. The story of how Ramanujan discovered his famous partition congruences upon seeing this table is also described.*

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## 5. Fermat and Ramanujan: A comparison

**Abstract:** *The famous French mathematician Pierre Fermat, like Ramanujan, made notes of his observations without proofs, and communicated his findings in letters to contemporaries. In this article a comparison is made of certain mathematical contributions of Fermat and Ramanujan, and of their mathematical tastes. The famous Ramanujan taxi cab equation is discussed as a Diophantine equation in four variables having solutions, but this equation becomes the cubic version of Fermat's Last Theorem when one of the variables is set equal to zero, in which case there are no non-trivial solutions.*

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## 6. J. J. Sylvester: Ramanujan's illustrious predecessor

**Abstract:** *Until the emergence of Ramanujan in the early twentieth century, the two great figures in the theory of partitions were Leonhard Euler, the founder of the subject, and J. J. Sylvester, who in the latter part of the nineteenth century, systematically improved Euler's results on partitions by a combinatorial approach. After describing the unusual life of Sylvester first at Cambridge University, England and later at Johns Hopkins University, USA, the article discusses some fundamental results of Sylvester, and how some of these on partitions relate to Ramanujan's contributions.*

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## 7. Erdős and Ramanujan: Legends of twentieth century mathematics

**Abstract:** *The Hungarian Paul Erdős was one of the greatest mathematicians of the twentieth century, and one of the most prolific in history. This article describes the unusual itinerant life of Erdős, his idiosyncrasies, and his life's mission to spot budding young mathematicians and encourage them. Erdős was inspired by Ramanujan's work on prime numbers and arithmetical functions. The origins of probabilistic number theory founded by Erdős and Marc Kac in the 1940s can be traced to the fundamental 1917 paper of Hardy and Ramanujan on round numbers, and the remarkable story of how the Erdős-Kac collaboration came about is also told.*

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## 8. C. G. J. Jacobi: Algorist par-excellence

**Abstract:** *The remarkable powers of C. G. J. Jacobi can be best seen in the theory of theta functions of which he was the founder. Ramanujan has been compared to Jacobi and Euler for sheer manipulative ability. This article describes the life of Jacobi and the monumental contributions he made even though he was plagued with health problems. A discussion of Jacobi's seminal work on theta functions is included, and a comparison is drawn with Ramanujan's own theory of theta functions.*

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## 9. Evariste Galois: Founder of group theory

**Abstract:** *The French mathematician Evariste Galois, had a tragic untimely death in a duel at the age of twenty, but had in his all too brief life, made a revolutionary contribution, name the founding of group theory. In commenting on Ramanujan's demise at the early age of 32, Hardy compared the Indian genius to mathematicians like Galois who died very*

young, yet had done their best work in their teens. This article describes the remarkable but tragic life of Galois, his monumental creation of group theory, and the relation of the work of Galois to Ramanujan's results on roots of numbers.

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#### 10. Leonhard Euler: Most prolific mathematician in history

**Abstract:** *With the exception of Ramanujan, the Swiss mathematician Leonhard Euler was unrivalled in the manipulation of infinite series. Euler laid the foundations of the theory of partitions where Ramanujan excelled by the use of identities for infinite series and products. After describing the life of Euler, a selection of some of his great contributions are discussed some of which have strong links with the work of Ramanujan, such as the theory of partitions and  $q$ -hypergeometric series, the Ramanujan taxi cab equation, and sums involving the roots of unity.*

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#### 11. G. H. Hardy: Ramanujan's mentor

**Abstract:** *The British mathematician G. H. Hardy was the consummate Cambridge professor. Upon seeing Ramanujan's letter he concluded that Ramanujan was a true genius and invited him to Cambridge. Here we describe Hardy's life story, his views about mathematical research and education, his encouragement of Ramanujan and their great collaboration.*

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#### 12. J. E. Littlewood

**Abstract:** *J. E. Littlewood was G. H. Hardy's close collaborator. Hardy consulted Littlewood to evaluate the significance of the claims in Ramanujan's letters. This article describes the life of Littlewood and various unique aspects of the Hardy-Littlewood partnership. Also included is a discussion of the Circle Method that Hardy and Littlewood developed into a powerful tool in additive number theory, the method having its basic formulation in the Hardy-Ramanujan paper on the asymptotics of the partition function.*

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#### 13. Niels Henrik Abel: Norwegian mathematical genius

**Abstract:** *Niels Henrik Abel, the Norwegian genius, like Ramanujan, died very young, but had made monumental contributions nonetheless. And like Ramanujan, Abel suffered from poverty and ill health, but did not let this impede his great creativity. We describe Abel's life making comparisons with Ramanujan's, and discuss certain aspects of Abel's work, such as his treatment of divergent series, and his work on elliptic functions, that have relationship with some of Ramanujan's discoveries.*

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#### 14. Issai Schur: Ramanujan's German contemporary

**Abstract:** *Cut off from England due to the First World War, the German mathematician Issai Schur independently discovered and proved the Rogers-Ramanujan identities and like MacMahon, realized their partition theoretic significance. Schur is most well known for his work in group theory, but his proof of the Rogers-Ramanujan identities has resulted in*

significant developments in the theory of partitions. Here we describe Schur's life and work both in group theory and on partitions, and make comparisons with the work of Ramanujan.

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#### **15. Robert Rankin: Scottish link with Ramanujan**

**Abstract;** *The Scottish mathematician Robert Rankin was associated with Ramanujan's mentor G. H. Hardy. He also handled the manuscripts in the estate of G. N. Watson, and had them deposited in the Wren Library at Trinity College, Cambridge University. Ramanujan's Lost Notebook was among these manuscripts. After describing Rankin's life, the remarkable story of the discovery of the Lost Notebook is recalled. The article also includes a discussion of Rankin's work in number theory relating to prime numbers, modular forms and the Ramanujan tau function.*

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#### **16. Ramanujan and $\pi$ :**

**Abstract:** *The number  $\pi$  is one of the most fundamental, and many great mathematicians have made important contributions to our understanding of this number. Ramanujan established amazing series representations for  $\pi$ . We describe the history of  $\pi$  starting from the days of Greece and ending in the modern world of the computer. We discuss how some of Ramanujan's series are used nowadays to calculate millions of digits of  $\pi$ .*

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#### **17: Ramanujan and partitions**

**Abstract:** *The theory of partitions founded by Euler in the mid-eighteenth century, underwent a glorious transformation under the magic touch of Ramanujan in the early twentieth century. The discussion includes the revolutionary work of Ramanujan on partition congruences, the Hardy-Ramanujan asymptotic formula for the partition function, and the Rogers-Ramanujan identities, and story of their discoveries. Also included is a discussion of the current state of research on partitions and the continuing influence of Ramanujan's ideas in this area.*

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#### **18: Major progress on a problem of Ramanujan**

**Abstract:** *The theorem of Lagrange that every positive integer is a sum of at most four squares, is one of the most appealing in all of mathematics. Ramanujan considered extensions of this theorem by listing a set of universal quadratic forms, namely those which represent all positive integers. This article is a report of the work of Manjul Bhargava and Jonathan Hanke who solved a conjecture of Conway of determining the universality of quadratic forms - a problem that has its origins in Ramanujan's work.*

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#### **19. Review of "Srinivasa Ramanujan: The Lost Notebook and other unpublished papers"**

**Abstract:** *This is a review of "Srinivasa Ramanujan: The Lost Notebook and other unpublished papers", which was printed in 1987 and released on December 22 of that year in Madras on the 100-th birth anniversary of Ramanujan.*

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**20. A review of “The Man Who Knew Infinity: A life of the genius Ramanujan”**

**Abstract:** *This is a review of Robert Kanigel’s highly influential biography on Srinivasa Ramanujan, the first detailed account of his most remarkable life.*

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**21. A review of “Ramanujan: Letters and Commentary”**

**Abstract:** *This is a review of the book “Ramanujan: Letters and Commentary” by Bruce Berndt and Robert Rankin, which analyzes in detail from a mathematical, historical, and social point of view, letters written to, from and about Ramanujan.*

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**22: A review of “Ramanujan: Essays and Surveys”**

**Abstract:** *This is a review of the book “Ramanujan: Essays and Surveys” by Bruce Berndt and Robert Rankin, which contains a selection of some of the most important articles written about Ramanujan’s life and work, and about the life of some individuals who played a crucial role in Ramanujan’s life.*

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**23: A review of “Partition: A Play on Ramanujan”**

**Abstract:** *This is a review of the play “Partition” written by Ira Hauptman, dealing with life of the genius Ramanujan, how he came to England to work with G. H. Hardy, and how the two collaborated to produce a most remarkable formula for the partition function.*

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**24. The Ramanujan Journal: Its conception, need and place**

**Abstract:** *Srinivasa Ramanujan’s discoveries continue to have a strong influence on various fields within and outside of mathematics. The launching of The Ramanujan Journal in 1997, devoted to all areas of mathematics influenced by Ramanujan, is a major event in the world of Ramanujan. The growth of this journal is a testimony that the mathematical legacy of Ramanujan is thriving. This article describes how this journal was conceived, why it fills a need, and the place it occupies in the world of mathematics.*

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**25: A pilgrimage to Ramanujan’s hometown**

**Abstract:** *The purchase of Srinivasa Ramanujan’s home in Kumbakonam by SASTRA University on 2003, is a major step in the preservation of Ramanujan’s legacy for posterity. To mark that occasion SASTRA University organized an international conference. The author’s visit to Kumbakonam for this conference began a series of annual visits. This article describes the cultural and mathematical legacy of Kumbakonam and provides an insight into the cultural setting in which Ramanujan grew up and made his most awe inspiring discoveries.*

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## 26. The First SASTRA Ramanujan Prizes

**Abstract:** *The SASTRA Ramanujan Prize, launched in 2005, is an annual prize of US \$ 10,000 given to a mathematician not exceeding the age of 32 for outstanding contributions in areas influenced by Ramanujan. The prize is now one of the most prestigious and coveted international awards. This is the speech given by author in 2005 as Chair of the Prize Committee, when the First SASTRA Ramanujan Prizes were awarded, describing why such a prize was conceived, and how it will play a major role in the encouragement of young mathematicians. The work and accomplishments of the winners are also described.*

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## 27. Ramanujan's growing influence

**Abstract:** *In the first half of the twentieth century, the only source to understanding Ramanujan's work were Hardy's lectures, besides the papers of Ramanujan. Students today are much more fortunate in that there are several books about Ramanujan's life, the mathematics in his Notebooks, and about the work in his research papers. Now there is also a journal devoted to all areas of mathematics influenced by Ramanujan. With Ramanujan's mathematics continuing to play a major role in current research, this article describes how these books and the journal would be helpful in attracting students to a study of Ramanujan's work, and how they could be used to offer courses at school and college levels.*

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