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(20b) GÖTTINGEN, 8.10.1956
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Dr. Alladi Ramakrishnan
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University of Madras
A.C. College Buildings,

M a d r a s - 25, (Indien)

Dear Dr. Ramakrishnan,

Many thanks for your letter and for your interesting manuscript on cascade theory. Your paper has been studied by the theoreticians of the institute, and we agree with you that your new approach may simplify the comparison between theory and experiment. From the view point of the experiments we would be very interested to have numerical calculations on the fluctuations of cascade showers of very high primary energy (above 20 GeV) at small thicknesses (t=0,5 - 2,0).

You would certainly be welcome for a visit of a few months in our institute during the next summer, and the physicists especially of our experimental group would be glad to discuss the problems of electronic cascade with you. One could scarcely speak of a lecturership in this connection because the Max-Planck-Institute is a pure research institution in which lectures to students are not given (it is not connected with the university), but you would be completely free to work in the institute in the same way as the other visiting scientists from abroad.

Yours very sincerely,

W. Heisenberg

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Probability and Stochastic Processes.

By

ALLADI RAMAKRISHNAN.

Prefatory note.

This essay needs an apology rather than a preface. It is an attempt to present to the physicist a physical approach to the theory of stochastic processes, a field till recently the close preserve of the mathematician. The only justification for the style and form adopted here lies in that the theory of stochastic processes as formulated in abstract mathematical treatises and papers is difficult to read even to those who are trained to a rigorous mathematical discipline. But in many problems where some random element is introduced, the physicist needs a knowledge of the results of stochastic theory and he would like to use them without being diverted by mathematical details or trammelled by the demands of rigour. Such examples are cited, but no pretence is made to completeness, and the emphasis is laid only on the methods used in applying a general theory to particular problems.

This article would have justified its purpose if it furnishes the physicist with such mathematical tools as are already available. It is hoped that incidentally it will persuade pure mathematicians who are inclined to look upon the subject of probability and stochastic processes as just a branch of measure theory to take a lively interest in the fields in which stochastic theory is applied.

Introduction.

The theory of stochastic processes is a natural extension of the theory of probability to dynamical problems, the word "dynamical" being used in its most general sense to denote the changes with respect to a parameter or a set of parameters. While probability theory and dynamics are old and well established branches of science, almost treated as classical, the theory of stochastic processes is of comparatively recent origin. The abstract mathematical formulation of the theory of stochastic processes has attracted the attention of many mathematicians. But somehow, the application to physical problems has been delayed. This delay can be ascribed not merely to the inadequate acquaintance of the physicists with the mathematical formalism but to a deeper and more fundamental reason —the difficulty of translating the physical conditions of the problems into a proper mathematical formulation. The object of the present contribution is merely to present stochastic theory in a serviceable form which can be applied to physical problems by a physicist. No attempt is made to intrude into the abstract field of pure mathematical theory which is for example found in the classic work of DOOB [1] and others. In presenting a physical approach we draw attention also to the correspondence with the abstract formalism. Incidentally we emphasise that in the very task of transcribing the physical conditions into a rigorous formulation, new concepts come to light which would otherwise escape attention if a purely logical and axiomatic approach is made.

The essay is divided into two parts. Part A is a brief and rapid survey of probability theory, a refresher course on a subject which is too well known to demand any repetitive discussions. But it is presented in a manner that leads directly to the concept of stochastic processes. In addition, the standard techniques used in probability theory are brought together in a compact form to serve as a ready reference. In this part of the essay, proofs or elaborate discussions are not given but the implications and scope of the fundamental theorems of probability are explained.

In Part B which deals with stochastic processes, we begin with a physical approach which is followed by a short chapter establishing the correspondence with the abstract formulation of DOOB. As many physical examples as the author could lay his hands on to illustrate stochastic processes are cited. Since modern physics is almost identified with quantum mechanics, the role of the theory of stochastic processes in physics is closely connected with that of quantum mechanics. Hence an attempt is made to point out the stochastic aspects of some processes in quantum mechanics. In this task, the author has been encouraged by the proceedings of a seminar course on probability problems in physics organised by LOEVE [2] and his associates and secondly by the scheme of subjects discussed by the probability seminar group at the M.I.T.

The first book which is an explicit attempt at the application of stochastic theory to physical problems is that of my teacher Professor BARTLETT [3] and this work is intended as a sequel to it. Hence the subjects dealt with in detail by him are not discussed here at length but referred to when necessary.

A. Probability.

I. Probability and measure theory.

Probability is an accepted branch of mathematics and it is but natural that mathematicians should have formulated it without any appeal to auxiliary concepts like experiments, trials, events and their occurrences. As DOOB [1] has emphasised, "such a formulation is necessary to avoid spurious simplifications of some parts of the subject and a genuine distortion of all of it". However, since the theory of probability and stochastic processes finds direct application throughout the field of physics and other branches of science, it is equally necessary to establish a clear correspondence between the physical nature of a problem and the mathematical abstraction of it. Thus we shall use intuitive concepts like experiments, events and trials, but take care to show how these can be carried over to the notions of the theory of sets and measure to which probability theory really belongs.

1. Events and probabilities. We assume that the outcome of an experiment $\mathcal{E}$ is one of a mutually exclusive set of n events, $E_1, E_2, E_3, \ldots, E_n$. For brevity the set is denoted by $\langle E_i \rangle$ [1]. We shall consider n to be finite and later extend it to the enumerable and continuous infinite cases. To each event E_k we assign a number $P(E_k)$ representing the probability that event E_k is the outcome. Before we ascribe a meaning to this number, we assume that without loss of generality, it is possible to set

$$\sum_k P(E_k) = 1. \tag{1.1}$$

The decomposition of the outcome into n mutually exclusive events is not necessarily unique. An event E', say, may denote either the event E_k or E_l and in such a case, we write symbolically

$$E' = E_k + E_l \tag{1.2}$$

and to E' we assign a number $P(E')$ the probability that E' i.e. E_k or E_l is the outcome of the experiment.

$$P(E') = P(E_k) + P(E_l). \tag{1.3}$$

Thus in the aggregate of mutually exclusive events $E_1, E_2, \ldots, E_n$ we may remove E_k and E_l and replace them by E' and now we have another set of $n-1$ mutually

[1] We shall in this essay denote aggregates by the symbol $\langle\,\rangle$. The members of the aggregate can be inferred from the context through the symbol used inside the bracket.