CONTRIBUTIONS OF PROFESSOR ALLADI RAMAKRISHNAN TO THE MATHEMATICAL SCIENCES

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Professor Alladi Ramakrishnan, my father, belongs to a small eminent group of Indian scientists who have made fundamental contributions to several fields of study and sustained a high level of productivity over a significant period of time. If among this select versatile group of researchers we seek those who also have made monumental contributions to the profession by creating leading institutions of advanced research, then we are down to a mere handful such as Professors Raman, Bhabha, Mahalanobis, Ramakrishnan, and a few more. In an illustrious scientific career that began in 1947, Professor Ramakrishnan has published over 150 influential research papers in leading journals on topics ranging over Stochastic Processes, Elementary Particle Physics, Matrix Algebra, and the Special Theory of Relativity, has guided about 30 PhD students, lectured on his research at over 200 institutions of higher learning the world over and at numerous international conferences, and created MATSCIENCE, The Insitute of Mathematical Sciences in Madras. As he crosses the milestone age of 80, I am amazed that his passion for science and his spirit of enquiry remains unabated. Here I shall briefly describe some of his significant contributions including his most recent ones, and the circumstances that led to them.

Early life and career choice: Right from his school days my father demonstrated his originality both in mathematics and physics. In Loyola College, Madras, he was awarded a special prize by his mathematics teacher Adivarahan who was much impressed by my father's unusual originality in classical geometry. I witness this facility with geometry even today, when my father applies simple but ingenious geometrical arguments to explain and unravel new features of difficult concepts in Special Relativity.

My paternal grandfather Sir Alladi Krishnaswami Iyer was one of the greatest lawyers of India during the first half of the twentieth century. He played a crucial role in drafting the Constitution of India. Naturally, to young Ramakrishnan, his father was a great influence. Indeed my father enrolled in law, passed the exams in flying colours, and secured a Gold Medal in Hindu Law. He even assisted my grandfather by taking notes to his dictation concerning the Indian Constitution. My father's early contacts through my grandfather were men of the calibre of Dr. S. Radhakrishnan, the great philosopher statesman who became the President of India, C. Rajagopalachari, and others. Inspite of all this exposure and contact with lawyers and statesmen, my father had this inner desire to pursue science as a career. Although my grandfather was immersed in the field of law, he used to say "compare the nationalism of politics to the internationalism of science," a sentence that profoundly influenced my father to change his career. Actually, the desire for my father to take to science as a career was kindled in 1943 when he heard a magnificent lecture on Meson Theory at the Presidency College, Madras, by Professor Homi Bhabha who had just returned to India from England as one of the youngest Fellows of the Royal Society (FRS). However it was only four years later after a brief stint in law that my father decided to eschew a lucrative legal career and take to science as a profession. It is at this instance that my paternal grandmother Lady Venkalakshmi convinced my grandfather to let her

young son pursue his dreams and goals.

Work with Bhabha: In 1947 my father joined what was at that time the fledgeling Tata Institute of Fundamental Research that functioned under Bhabha's direct supervision in Kenilworth, Bhabha's aunt's home. Thus my father was one of the first members of the Tata Institute who worked closely in contact with Bhabha himself. My father always stressed that the greatest gift a teacher can give a research student is a good problem, and in this sense he was very fortunate that Bhabha introduced him to Cascade Theory and the Fluctuation Problem of Cosmic Radiation. The study of this problem required the probabilistic analysis of the distribution of a discrete number of particles in continuous energy space. My father soon realised that it was possible to attack this problem directly by noting that the contribution to the density comes from the probability of a single particle in an infinitesimal interval which is proportional to the length of that interval, the coefficient representing the density. He named the correlation densities as *Product Densities*, a name that is still in vogue today. Bhabha, who was a master of limiting processes, had also an idea of how to solve this problem, but by a longer method.

Product Densities, and related work: In August 1949 my father left the Tata Institute and sailed to England with my mother to complete his PhD under the direction of Professor M. S. Bartlett at the University of Manchester. Professor Bartlett consulted his distinguished friend Professor D. G. Kendall who was then at Magdalene College at Oxford University. Professor Kendall not only confirmed the correctness of my father's work, but also approved the name product densities. Kendall had previously arrived at such functions up to the second order in his pioneering studies on population growth and called them cumulant densities. In my father's work on product densities, the more general $n-th$ order functions were considered. Thus, within two months of arrival in England, my father had completed his work for the PhD. But he had to stay for two years there to complete his residency requirements. My father's PhD work on product densities appeared in the Proceedings of the Cambridge Philosophical Society (1950) and Bhabha's alternate approach appeared around the same time in the Proceedings of the Royal Society.

Some years later my father had the opportunity to give a talk on applications of stochastic processes to cascade theory at the Max Planck Institute in Gottingen. The German Nobel Laureate Werner Heisenberg who heard this lecture made very complimentary comments. On the basis of Heisenberg's comments, Professor S. Flugge of Springer Verlag invited my father to write a comprehensive article on stochastic processes with emphasis on product densities. This authoritative article, the first of its kind on these topics, had a significant influence and resulted in a flood of papers in the area, most notably by Professor S. K. Srinivasan. A book by A.T. Barucha Reid on Markov Processes makes ample references to product densities and the work of Bhabha-Ramakrishnan. The method of product densities became very well known and is considered by many to be perhaps my father's most significant contribution. It is this work and its many applications by my father, his students, and other contemporaries in probability and stochastic processes, that has led to theme of this conference when this volume is being released.

In the 1950s, my father worked on the problem of the Fluctuating Density Field that came up in studies of the Milky Way by the great Indian astrophysicist Subramaniam Chandrasekhar. My father wrote a series of eight papers on this subject. Chandrasekhar

was so impressed, that he communicated all of them to the Astrophysical Journal.

Another notable contribution was my father's work on *Inverse Probability* in Stochastic Processes leading to the concept of the origin of a stochastic process. This paper was presented to the Indian Academy of Sciences in 1956. It was on the basis of this presentation that Sir. C. V. Raman had my father elected immediately as a Fellow of the Indian Academy of Sciences.

The work on inverse probability had other implications. It led my father to interpret the Feynman observation of a negative energy electron travelling back in time as actually tracing back in the inverse probability sense. This yielded a simple proof of the equivalence of the Feynman and the field theoretic formulation by splitting the Feynman propagator into positive and negative energy parts. The first person to establish this equivalence rigorously was Dyson, but only a few have really understood Dyson's deep and difficult derivation. My father's paper on this topic appeared in the Journal of Mathematical Analysis and Applications (1964). In addition, at the invitation of Professor Heitler, he published his work on stochastic processes and the Feynman propagator as a book entitled *Elementary particles and cosmic rays* published by the Pergamon Press (1963).

Visit to the Institute for Advanced Study: The year 1957-58 was another turning point in my father's career when he visited the Insitute for Advanced Study in Princeton at the invitation of its Director Robert Oppenheimer. At the Institute my father had the opportunity to listen to the lectures of, and discuss with, the leading young physicists of that generation, like T. D. Lee and C. N. Yang, who soon afterwards won the Nobel Prize in Physics. My father returned to India filled with the desire to induct talented students into theoretical physics and expose them to the latest advances in this field.

The Theoretical Physics Seminar: Not satisfied with the curriculum at the University of Madras where my father was a professor, he gave lectures on quantum mechanics and other advanced topics at our family home Ekamra Nivas during the period 1958-61 and named this the *Theoretical Physics Seminar*. As the daughter of a professor of mathematics Dr. H. Subramani Iyer, and as one who had accompanied my father to England and to Princeton, my mother Mrs. Lalitha Ramakrishnan had a full understanding of the significance of such efforts by my father. With enthusiasm she hosted the eager students who gathered in Ekamra Nivas for the Theoretical Physics Seminar, and the many eminent scientists who lectured at our home. Among the luminaries who addressed the students at the Theoretical Physics Seminar were Nobel Laureate Donald Glaser, and Professors Murray Gell Mann and Abdus Salam, both of whom won Nobel Prizes later.

Creation of MATSCIENCE: In 1960, Nobel Laureate Professor Niels Bohr visited India as the guest of Prime Minister Jawaharlal Nehru. When Bohr came to Madras, there was only one group of students who could understand his lectures, namely, those trained by my father. Bohr spent a leisurely evening at Ekamra Nivas discussing with my father and his students. When Bohr returned to Delhi, Nehru asked him what his impressions about science in India were. Professor Bohr said that two things impressed him most - the massive Tata Institute of Fundamental Research in Bombay, and the small group of students trained by Alladi Ramakrishnan in Madras! This statement by Bohr was flashed in the newspapers like The Hindu and sparked Nehru's interest to contact my father. Mr. C. Subramaniam arranged for a meeting at the Governor's Residence, Raj Bhavan, in

Madras, between Nehru and my father, in which the students of the Theoretical Physics Seminar were introduced to the Prime Minister. At this meeting Nehru asked my father what he wanted. Here was the Prime Minister of India asking you what you want. At such an instance, you do not ask for anything meagre. So my father asked for an institute for advanced fundamental research in the mathematical sciences like the Institute for Advanced Study in Princeton. The rest is history. With the recommendation of Niels Bohr, the support of C. Subramanian, and the benevolence of Jawaharlal Nehru, MATSCIENCE, The Institute of Mathematical Sciences was created in 1962 with my father as the Director. Subramaniam Chandrasekhar was invited to inaugurate the institute. I remember sitting in the English Lecture Hall of the Presidency College, Madras that day and listening to a magnificent lecture by my father - perhaps the finest he has delivered in his life thus far.

My father served as the Director of MATSCIENCE for 21 years until his retirement in 1983. He conceived it in his family home, nurtured it in its infancy, and saw it grow in size and stature. During his tenure as Director, hundreds of eminent mathematicians and physicists visited the Insitute, including Nobel Laureates Hans Bethe, Hans Jensen, Linus Pauling and John Bardeen, Fields Medallists Laurent Schwarz and Rene Thom, the mathematical giant Marshall Stone, the eminent Indian statistician C. R. Rao, the Ramanujan expert and partition authority George Andrews, and the legendary mathematician Paul Erdös. It was also during these 21 years that he travelled widely, lecturing at about 200 centers of learning all over the world. My mother and I accompanied him on these trips. The constant contact with outstanding academicians during these foreign tours as well as those who visited MATSCIENCE, and the experience of visiting several great centers of learning, made a deep impression on me. I thought of nothing else but an academic career and was naturally led into it.

Work in quantum mechanics: From the early sixties on, my father's research shifted to theoretical and particle physics. His most significant work in this area was the prescription he gave to make the transition from Pauli to Dirac Matrices. He called this the $\sigma - operation$. This work was part of a more comprehensive study of $\omega - commutation\ relations$ among matrices, generalizing the anti-commuting property of the Pauli matrices. All this occupied him and his students for about a decade, when a series of about 50 papers were published under the banner of *L-matrix theory*, mostly in the Journal of Mathematical Analysis and Applications. Subsequently these papers were also published collectively in the form of a book entitled *L-matrix theory or the grammar of Dirac matrices* by Tata McGraw Hill (1972) and released by His Excellency V. V. Giri, the President of India.

PhD students: My father's work on product densities and L-matrix theory provided food for thought for talented students who worked under his guidance. Over a period of a quarter century (1958-83), he produced about 30 PhD students. He provided opportunities for all of them to go abroad to visit centers of learning and to participate in international conferences. He was extremely generous in providing ample leave for them to travel, much to the envy of scientists in other institutions in India where leave and travel rules were much stricter. My father believed, and still believes, that young researchers would profit by contact with experts at institutions worldwide, and he therefore provided opportunities for them to travel. There was of course the risk of losing some of these talented students to other institutions. But he was convinced that science is an international enterprise

and therefore did not want the students to feel stifled due to lack of travel. Such large heartedness among senior administrators is hard to find. Some of the students who went abroad did not return, but made successful careers in the United States. Some others joined the faculty at MATSCIENCE. Four students accepted positions at educational institutions in Madras and all four not only developed schools of research at their respective institutions, but also became heads of their departments. They were Professors P. M. Mathews at the Department of Theoretical Physics of the University of Madras, S. K. Srinivasan of the Department of Mathematics at IIT Madras, V. Devanathan of the Department of Nuclear Physics of the University of Madras, and A. Vijayakumar of the Mathematics Department of Anna University, Madras.

Work in Special Relativity: My father had a fascination for Special Relativity since his college days inspired by the book of Joos on Theoretical Physics that he read at the suggestion of Sir C. V. Raman. His first significant piece of work in this area were a series of papers on the theme *Einstein is a natural completion of Newton* that appeared in the Journal of Mathematical Analysis and Applications. In a paper entitled *Ramakrishnan's approach to the theory of relativity* that also appeared in same journal (1974), the famous analyst Norman Levinson of MIT rigorously established some of the postulates my father made in his papers.

My father was always intrigued why only Einstein received the credit for the theory of relativity when so much of the theory depended on the Lorentz Transformation. In his years after his retirement in 1983, he has come back time and again to the Lorentz transformation, offering new and elegant derivations of it using simple but ingenious geometric arguments. He feels that although the Lorentz transformation is over a hundred years old, it still bears a youthful countenance. His work on the Lorentz transformation reached a peak in his paper *A rod approach to the theory of relativity* in which he clarified the distinction between Space-like and Time-like intervals. This paper appeared in the Special Millennium Issue of the Journal of Mathematical Analysis and Applications in September 2000 in honor of its Founding Editor Professor Richard Bellman with whom my father had close scientific contact since 1956.

Current training of students: Although my father retired twenty years ago, in these two decades he has continued to inspire and influence talented students. Especially in the last few years, several brilliant high school and undergraduate students have come to his home in Madras to learn from him. They have profited immensely by his instruction and encouragement, because every one of them has come to the United States to pursue higher studies in order to take to a career of research. I have met only one other person, the mathematical legend Paul Erdös, who had such a passion to meet talented young minds and encourage them to take to mathematics. My father's appetite for research and the desire to train students has not diminished with time. Thus, as he crosses this milestone age of 80, we wish him many more years of good health, so that he can continue to revel in the ecstasy of mathematics and inspire the younger generation to take to research in the mathematical sciences.

I close with the dictum that still dominates his life:

The pursuit of science is at its best
when it is a part of a way of life.